# Capacity Analysis of Spectrally Overlapping Direct-Sequence Spread Spectrum (DSSS) Channels


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**Abstract-** *It is well known that cellular business has been increasing exponentially in the last decade. With the huge increase in the number of cellular users, capacity of the existing cellular system has become an issue. In this paper, a novel approach is presented that provides for a significant increase in the number of users in multi-user DSSS systems.*


## I. Introduction

Using code division multiple access (CDMA) techniques, it is possible to have multiple users which simultaneously transmit information over a single channel. This is achieved by use of a different spreading code for each user. Ideally the spreading codes in a multi-user DSSS system are orthogonal. There is then zero interference between users provided that the data streams are synchronized. However, in practice, the spreading codes are only approximately orthogonal because the number of codes that are strictly orthogonal for a given length is very limited and, in addition, the data streams of multiple users are not likely to be synchronized. As a result, the variance of the decision statistic at the output of the correlation receiver is not zero. This variance ,which limits the number of users for a prespecified probability of error, is found to be [1]

$$\sigma^2 = \sum_{k=1}^{K_1-1} \int_{-\infty}^{\infty} [W_{s_k}(f) \otimes W_{v_o}(f)][T_{do}\text{sinc}(fT_{do})]^2 df \quad (1)$$

where $\otimes$ denotes convolution, $(K_1 - 1)$ is the number of interfering users, $T_{do}$ is the bit duration, and $W_{s_k}(f)$ and $W_{v_o}(f)$ are the power spectral densities (PSDs) of the transmitted DSSS signals and the correlating signal at the receiver, respectively, and are given by [1,2]

$$W_{s_k}(f) \approx \frac{T_{ck}P_k}{2}\{\text{sinc}^2[(f-f_{ck})T_{ck}]+\text{sinc}^2[(f+f_{ck})T_{ck}]\} \quad (2)$$

and

$$W_{v_o}(f) = \frac{T_{c_o}}{4}\{\text{sinc}^2[(f-f_{co})T_{co}]+\text{sinc}^2[(f+f_{co})T_{co}]\}. \quad (3)$$

$T_{ck}$, $P_k$, and $f_{ck}$ in Eq. (2) are the chip duration, average power, and carrier frequency of the $k^{th}$ DSSS interferer, respectively, while $T_{co}$ and $f_{co}$ in Eq. (3) are the chip duration and carrier frequency of the desired DSSS signal, respectively. Invoking the central limit theorem, the interference statistic is modeled as a Gaussian random variable. The probability of error is then given by [1,2]

$$P_e = Q\left(\sqrt{\frac{P_oT_{do}^2}{2\sigma^2}}\right) \quad (4)$$

where, by definition,

$$Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \quad (5)$$

and $P_o$ is the average power of the desired DSSS signal.

As seen from the interfering signal variance expression given in Eq. (1), analytical calculation of this variance is a very difficult task. The $k^{th}$ integral involves the convolution of $W_{sk}(f)$ with $W_{vo}(f)$, given by Eqs. (2) and (3), respectively, weighted by the square of a sinc function. Each of the power spectral densities in the convolution consists of the squares of sinc functions. Therefore, the integrand of the $k^{th}$ integral involves products of the square of sinc functions. Closed form evaluation of such an integral is not known. However, this difficulty can be overcome by upper-bounding the squared sinc terms in the variance integral with truncated cosine squared functions. Having made this approximation, the approximated PSDs of the transmitted DSSS signals can be expressed as

$$\hat{W}_{sk}(f) = \frac{P_k T_{ck}}{2}\{cos^2[(f+f_{ck})\frac{\pi T_{ck}}{2}]\Pi(\frac{f+f_{ck}}{2/T_{ck}}) + \cos^2[(f-f_{ck})\frac{\pi T_{ck}}{2}]\Pi(\frac{f-f_{ck}}{2/T_{ck}})\}. \quad (6)$$

where the rectangular function is

$$\Pi(\frac{f}{2\omega}) = \begin{cases} 1 & -\omega \leq f \leq \omega \\ 0 & \text{otherwise.} \end{cases} \quad (7)$$

Using the identical reasoning that led to the approximations in Eq. (6), $W_{v_o}(f)$ and $G(f) = [T_{do}\text{sinc}(fT_{do})]^2$ can be approximated in the same manner. The approximations for $W_{v_o}(f)$ and $G(f)$ are then given by

$$\hat{W}_{v_o}(f) = \frac{T_{c_o}}{4}\{\cos^2[(f+f_{co})\frac{\pi T_{c_o}}{2}]\Pi(\frac{f+f_{co}}{2/T_{co}}) + \cos^2[(f-f_{co})\frac{\pi T_{c_o}}{2}]\Pi(\frac{f-f_{co}}{2/T_{c_o}})] \quad (8)$$

and

$$\hat{G}(f) = T_{do}^2\cos^2(\frac{\pi f T_{do}}{2})\Pi(\frac{f}{2/T_{do}}). \quad (9)$$

Substitution of the approximations, defined above, into the $k^{th}$ term in Eq.(1), the variance due to the $k^{th}$ interferer is approximated by

$$\sigma_k^2 \approx \int_{-\infty}^{\infty}[\hat{W}_{s_k}(f) \otimes \hat{W}_{v_o}(f)]\hat{G}(f)df. \quad (10)$$

This is a general result that can be used whether or not the desired and interfering signals have the same carrier frequencies and bandwidths. Now this general result is utilized to determine the co-channel and adjacent channel interference variances , which are needed for the capacity analysis of spectrally overlapping DSSS channels.

Consider a case where a single interfering DSSS signal and the desired DSSS signal have identical carrier frequencies and bandwidths (i.e., $f_{ck} = f_{co}$ and $T_{ck} = T_{co}$). By using Eq.( 10) the co-channel interference variance is found to be

$$\sigma_{cc,1}^2 = \frac{T_{do}^2}{16}[ \frac{4L_o^6 + 4L_o^5\pi^2 - (6+\pi^2)L_o^4 - 8\pi^2 L_o^3 + 2\pi^2 L_o^2 + 4\pi^2 L_o + 4 - \pi^2}{2\pi^2 L_o^2(1-L_o^2)^2} + \frac{L_o^2(3-2L_o^2)}{\pi^2(1-L_o^2)^2}\cos(\frac{\pi}{L_o}) + \frac{L_o(1-2L_o)}{2\pi(1-L_o^2)}\sin(\frac{\pi}{L_o})]P_k \quad (11)$$

where $L_o = T_{do}/T_{co}$ is the processing gain.

For adjacent channel interference variance, consider the case where a single interfering DSSS signal has the same bandwidth as the desired signal but a carrier frequency spaced half a bandwidth from the desired signal carrier frequency (i.e., $f_{ck} = f_{co} + 1/T_{co}$ and $T_{ck} = T_{co}$). The adjacent channel interference variance is found to be

$$\sigma_{ac,1}^2(L_o) = \frac{T_{do}^2}{32\pi(L_o^2-1)L_o}[2\pi L_o^2 - 2\pi - L_o^3\sin(\pi/L_o)]P_k. \quad (12)$$

Having obtained the co-channel and adjacent-channel interference variances, attention is devoted to a novel technique which provides for a significant increase in the number of users in multiple-users DSSS systems. The technique is based on division of the DSSS channel into a number of sub-channels.

Consider a multiple user DSSS system assigned to a single channel of bandwidth, $B_1$. The maximum processing gain for each user is achieved by spreading its spectrum to fill the entire channel bandwidth. Consequently, in multiple user DSSS systems, each user has maximum bandwidth, $B_1$, and the same carrier frequency, $f_{co}$, located at the center of the channel. For simplicity, assume each user has the same average power, $P_o$. Given the processing gain and the prespecified probability of error, it is possible to determine the maximum number of users. We refer to this case as having a full channel spectral allocation and denote the maximum number of users by $K_1$. For the same channel of bandwidth, $B_1$, the question arises as to whether it is possible to allocate each user's spectrum in a manner different from the full channel allocation so as to increase the number of users beyond $K_1$.

## II. Full Channel Spectral Allocation

In order to establish the notation to be used, the full channel spectral allocation case is discussed first. This case serves as the base line. Here all users have the same bandwidth equal to the channel bandwidth, $B_1$, and the same center frequency, $f_{co}$. In addition, each user is assumed to have the same average power, $P_o$. The maximum number of users is denoted by $K_1$. This number is evaluated under the condition that the same prespecified probability of error for each user is not exceeded.

Since the bandwidth of a rectangular pulse is equal to the inverse of its pulse width, the processing gain can be also expressed as

$$L_1 = \frac{B_1}{B_d} \quad (13)$$

where $B_1$ and $B_d$ are the bandwidths of the chip and data pulses, respectively. The variance due to a single interferer is given by Eq. (11). Because the interferers have the same average power and processing gain, the variance due to the $(K_1 - 1)$ interferers is

$$\sigma^2_{cc,(K_1-1)}(L_1) = (K_1 - 1)\sigma^2_{cc,1}(L_1) \tag{14}$$

where statistical independence between the users is assumed. The maximum number of users is determined by increasing $K_1$ to its maximum value such that the prespecified probability of error is not exceeded.

## III. Overlapping Sub-channels

Now assume that the entire channel of bandwidth $B_1$ is divided into $m$ non-overlapping sub-channels of bandwidth $B_m = B_1/m$ and $(m-1)$ overlapping sub-channels having the same bandwidth $B_m$ but centered at the cut-off frequencies of the non-overlapping sub-channels. Therefore, a total of $(2m - 1)$ sub-channels of bandwidth $B_m$ subdivide the original channel of bandwidth $B_1$. Fig. (1) illustrates the case for which the channel of bandwidth $B_1$ is subdivided into four non-overlapping sub-channels (m=4) plus three (m-1=3) additional sub-channels to produce 7 overlapping sub-channels. As

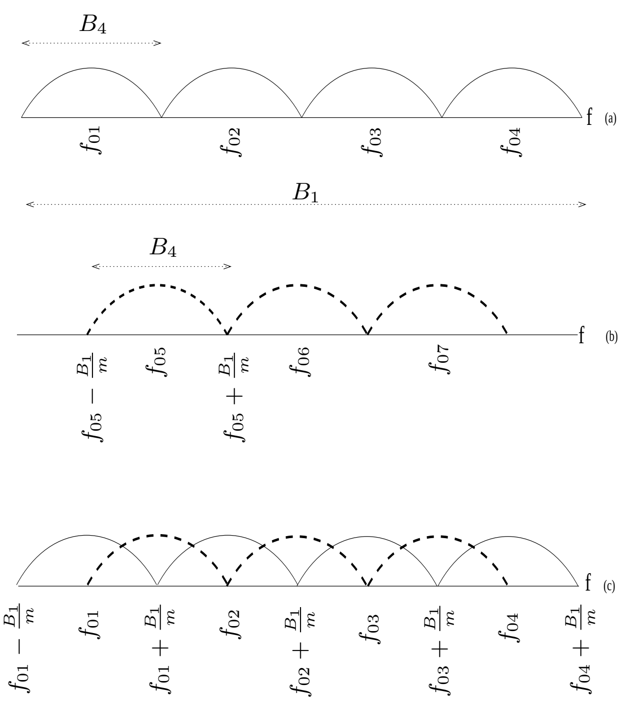


Figure 1: The case for which m=4 (a) 4 non-overlapping sub-channels (b) 3 additional sub-channels (c) 7 overlapping sub-channels

can be seen from Fig. (1), a desired signal now experiences interference from both other users in the same subchannel (i.e., co-channel interference) as well as those in the neighboring sub-channels (i.e., adjacent channel interference). For simplicity, a worst case situation is analyzed where it is assumed that all desired users suffer adjacent channel interference from the sub-channels on both sides even though the end-channels experience interference from only one side. Once again, the objective is to maximize the number of users such that the probability of error experienced by each user does not exceed the common prespecified probability of error. Using symmetry and neglecting end-channel effects, the maximum number of users allowed within each sub-channel is equal and is denoted by $K_{2m-1}$. Here, the subscript on $K$ represents the presence of (2m-1) sub-channels within the original channel of bandwidth $B_1$. It follows that there are $(K_{2m-1} - 1)$ co-channel interferers and $2K_{2m-1}$ adjacent-channel interferers. Therefore, the total interference variance is

$$\sigma^2_{(tot)}(L_m) = (K_{2m-1} - 1)\sigma^2_{cc,1}(L_m) + 2K_{2m-1}\sigma^2_{ac,1}(L_m) \tag{15}$$

where $L_m$ is the sub-channel processing gain. Observe that each sub-channel has the same bandwidth, $B_m$. Consequently, the processing gain for each DSSS users is equal and is given by

$$L_m = \frac{B_m}{B_d} = \frac{B_1/m}{B_d} = \frac{L_1}{m}. \tag{16}$$

It is of interest to plot the ratio between the variances in Eqs. (11) and (12) as a function of $L_m$. In particular, let

$$R_{\sigma^2} = \frac{\sigma^2_{cc,1}(L_m)}{\sigma^2_{ac,1}(L_m)}. \tag{17}$$

This ratio is plotted in Fig. 2. From this figure, it is seen

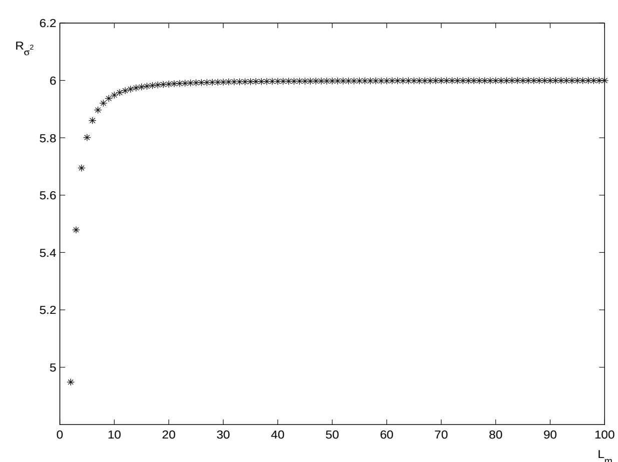


Figure 2: Plot of the ratio $R_{\sigma^2}$ vs. $L_m$.

that $R_{\sigma^2} \approx 6$ for $L_m \geq 10$.

Assuming $L_m \geq 10$,

$$\sigma^2_{ac,1}(L_m) \approx \frac{1}{6}\sigma^2_{cc,1}(L_m) \quad (18)$$

Use of Eq. (18) in Eq. (15) yields

$$\begin{aligned} \sigma^2_{(tot)}(L_m) &= (K_{2m-1}-1)\sigma^2_{cc,1}(L_m) \\ &+ \frac{2}{6}K_{2m-1}\sigma^2_{cc,1}(L_m) \\ &= (\frac{4}{3}K_{2m-1}-1)\sigma^2_{cc,1}(L_m). \quad (19) \end{aligned}$$

Recall that $K_1$ denotes the maximum number of users for the single channel case such that the prespecified probability of error for each user is not exceeded. So that this condition on the probability of error holds for the present case of overlapping sub-channels it is necessary that

$$\begin{aligned} \sigma^2_{(tot)}(L_m) &\leq \sigma^2_{cc,(K_1-1)}(L_1) \\ (\frac{4}{3}K_{2m-1}-1)\sigma^2_{cc,1}(L_m) &\leq (K_1-1)\sigma^2_{cc,1}(L_1) \\ K_{2m-1} &\leq \frac{3}{4}[(K_1-1)\frac{\sigma^2_{cc,1}(L_1)}{\sigma^2_{cc,1}(L_1/m)}+1]. \quad (20) \end{aligned}$$

This requires that the ratio $\sigma^2_{cc,1}(L_1)/\sigma^2_{cc,1}(L_1/m)$ be known. Determining this ratio requires evaluation of Eq. (11) for $L_1$ and $L_1/m$. An analytical expression for $\sigma^2_{cc,1}(L_1)/\sigma^2_{cc,1}(L_1/m)$ is very cumbersome. Hence, $\sigma^2_{cc,1}(L_1/m)/\sigma^2_{cc,1}(L_1)$ is plotted versus m in Fig. 3 to get an idea of how this ratio varies with m. Note from Fig. 3 that the relationship is approximately a straight line with $45^o$ slope. Hence,

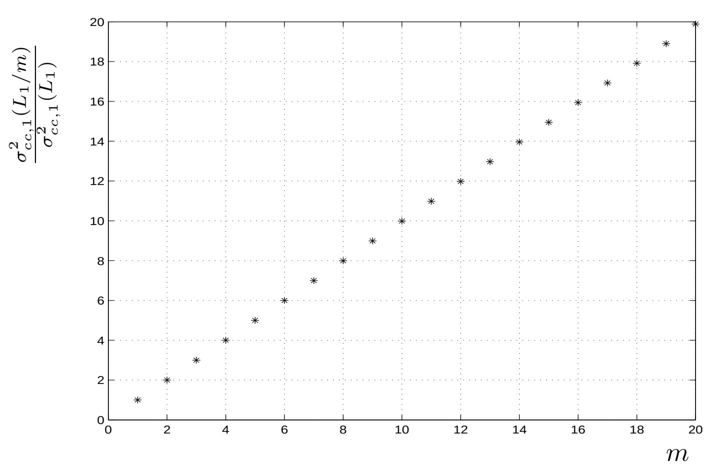


Figure 3: Plot of the ratio $\sigma^2_{cc,1}(L_1/m)/\sigma^2_{cc,1}(L_1)$ as a function of $m$

$$\frac{\sigma^2_{cc,1}(L_1/m)}{\sigma^2_{cc,1}(L_1)} \approx m. \quad (21)$$

Substitution of Eq. (21) into Eq. (20) yields the conclusion that $K_{2m-1}$ is the maximum integer such that

$$K_{2m-1} \leq \frac{3}{4}[\frac{(K_1-1)}{m}+1]. \quad (22)$$

Since there are (2m-1) sub-channels, the maximum number of users is the largest integer such that

$$\begin{aligned} K_{T,(2m-1)} &= (2m-1)K_{2m-1} \\ &\leq \frac{3}{4}\frac{(2m-1)}{m}[K_1-1+m]. \quad (23) \end{aligned}$$

To verify the analytical results and to gauge the accuracy of the simplified worst-case approach, a Monte Carlo computer simulation was carried out. Due to limitations of the computing resources, the simulation assumes that the processing gains and frequencies used for the simulation are smaller than those used in actual practice. Fig. 4 depicts a DSSS channel of bandwidth $B_1 = 12$ kHz. It is assumed that the processing gain is $L_1 = 60$ and there are a total of 78 users for the simulation. The probability of error for this case obtained from the computer simulation is 0.064. Letting m=3, the channel of Fig. 4 was subdi-

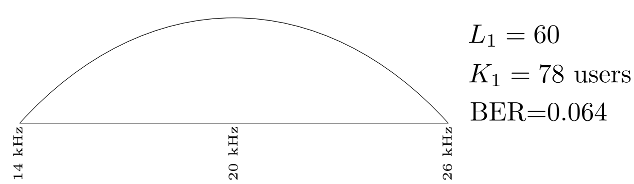


Figure 4: DSSS channel before subdivision

vided into five overlapping sub-channels of equal bandwidth as shown in Fig. 5. With reference to Eq. (23), the

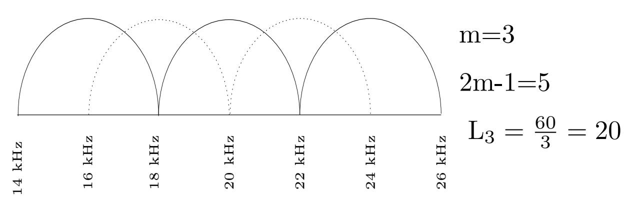


Figure 5: DSSS channel after subdivision when m=3

maximum number of users that can be allocated within each sub-channel is the largest integer such that

$$\begin{aligned} K_5 &\leq \frac{3}{4}[\frac{(78-1)}{3}+1] \\ &\leq 20. \quad (24) \end{aligned}$$

Hence, $K_5 = 20$. Using a monte Carlo simulation, the probability of error with 20 users for each sub-channel was found to be 0.0544. The smaller probability of error is believed to be due to the worst case approach used in the analysis. The total number of users within the entire channel is the sum of the users in the individual sub-channels. Therefore, there will be a total of 5x20=100 users. When the channel was not subdivided, the number of users was 78. This is an increase of 28.3%. As shown in Fig. 6, this procedure was repeated for m=4. Making

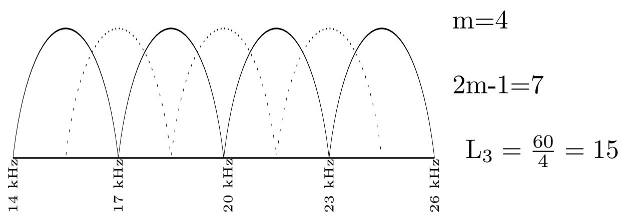


Figure 6: DSSS channel after subdivision when m=4

use of Eq. (23), $K_{2m-1} = K_7$ is the largest integer such that

$$K_7 \leq \frac{3}{4}[\frac{(78-1)}{4} + 1] = 15.1875 \quad (25)$$

Thus, $K_7 = 15$. Using a Monte Carlo simulation, the probability of error is found to be 0.0413. Once again, the effect of our worst-case approach is seen. This yields an increase in the number of users by 34.6 % even with our worst-case approach. Table (1) summarizes the results for m=1,3,4,5, and 6. Note that the percentage increase in users grows with m. The limiting case with increasing $m$ occurs for $m = 60$. This results in 119 users or an increase of 52.69 %.

# IV. Conclusion

As the number of users is increased, performance of the conventional DSSS system becomes an issue. Hence, we proposed a novel approach that can provide a significant increase in the number of users in a given bandwidth over that presently available. The proposed technique is based on division of the entire channel into a number of sub-channels having a smaller bandwidth than the entire channel bandwidth. Analytical results were obtained to estimate the number of users when the channel is divided into smaller bandwidth sub-channels. It is seen from the analytical results and computer simulations that the proposed approach allows more users to be accommodated than is now possible with the conventional single carrier DSSS system.

Table 1: Increase in Number of Users for Overlapping Sub-channels

| m | $L_m = \frac{L_1}{m}$ | $2m-1$ | $K_{2m-1}$ | $(2m-1)K_{2m-1}$ | BER | % Improvement |
|---|---|---|---|---|---|---|
| 1 | 60 | 1 | 78 | 78 | 0.0640 | 0 |
| 3 | 20 | 5 | 20 | 100 | 0.0544 | 28.1 |
| 4 | 15 | 7 | 15 | 105 | 0.0413 | 34.6 |
| 5 | 12 | 9 | 12 | 108 | 0.0614 | 38.4 |
| 6 | 10 | 11 | 10 | 110 | 0.0531 | 41.1 |

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